PROPERTIES OF FET PARAMETER STATISTICAL DATA BASES

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ABSTRACT
Statistical data bases are often used to characterize the statistics of a FET. This paper shows that a data base containing FET model parameter marginal probability density functions and covariance matrix is not sufficient to describe the FET's S-parameter statistics. This result is important to those developing statistical data bases for GaAs FETs. The implications of this work to simulation and CAD are discussed and a solution to this problem, the Truth Model, is presented.

1.0 INTRODUCTION

Generally a FET's RF performance is characterized by its S-parameter measurements over frequency and bias. These measurements are transformed into a compact model representation using an optimization based fitting algorithm. This modeling represents a nonlinear transformation from S-Parameters (SP) to FET Parameters (FP). Due to variability in the manufacturing process, the SP's and the FP's are actually random variables described by joint Probability Density Functions (PDF). The statistics for the FP's are "measured" by transforming the SP measurements from many FETs and then estimating the FP's statistical properties. In general, if the joint PDF of the FP's is determined, the joint PDF of the SP measurements has been captured.

The joint PDF for the FP's is difficult to estimate from measurements. Usually only the marginal PDF's and the COVariance (COV) matrix are determined. However, for arbitrarily distributed SP's, the joint PDF of the SP's is not recoverable when given only the marginal PDF's and COV of the FP's [1]. This is even true for Gaussian SP's due to the nonlinear transformation between the SP's and FP's. Therefore the question arises: Given the marginal PDF's and COV of the FP's, is this sufficient information to reproduce the marginal PDF's and COV of the SP's? We show using measured FET data that the answer is no. This result should be of interest to those who are developing statistical data bases for GaAs FET's and to those who use FET statistics in simulation and CAD. To illustrate the principals involved, we first show a simple example.

2.0 SIMPLE EXAMPLE

Suppose S1 and S2 are two measured variables. They are independent and uniformly distributed over ($-1<S1<1$, $-1<S2<1$). We transform the variables S1 and S2 into two model parameters P1 and P2 where $P1 = S1*\cos(\pi*S2)$ and $P2 = S1*\sin(\pi*s2)$. This transformation is nonlinear and invertable. The COV(P1, P2) is the identity matrix.

One thousand samples of S1 and S2 were simulated and then transformed into P1 and P2. We call these $P1_o$ and $P2_o$ (for "original"). Then one thousand independent samples of P1 and P2, called $P1_s$ and $P2_s$ (for "simulated"), were simulated using the marginal densities of $P1_o$ and $P2_o$. A good method to compare two densities is the nonparametric Kolmogorov-Smirnov (K-S) two sample test [5]. This test gives the probability that two data sets have come from the same PDF. The Kolmogorov-Smirnoff (K-S) test on $P1_o$ and $P1_s$ is .7 and the K-S test on $P2_o$ and $P2_s$ is 1.0. Hence the original and simulated densities are statistically the same with confidence of at least .7. The COV ($P1_o$, $P2_o$) = .004 and the COV($P1_s$, $P2_s$) = .005. This data shows that the "low order" statistics on $P1_o$,$P2_o$ and $P1_s$,$P2_s$ are statistically the same. Figure 1 shows a scatter plot for $P1_o$,$P2_o$ and $P1_s$,$P2_s$. The constraints of the transformation require that the scatter plot points for $P1_o$,$P2_o$ lie within the circle of unit radius, and the scatter plot data shows this. However the $P1_s$,$P2_s$ scatter plot shows no such bound. This difference in the data sets $P1_s$,$P2_s$ and $P1_o$,$P2_o$ is not detected by the marginal density or the covariance of the data sets.

The point illustrated here appears obvious; two statistically different multivariate data sets can have the same "low order" statistics; like marginal density and covariance. The reader may say, "They look similar to me. What's the problem?" The problem arises when the data is nonlinearly transformed. The same nonlinear transformation applied to two data sets with the same low order statistics can result in transformed variables with "low order" statistics that

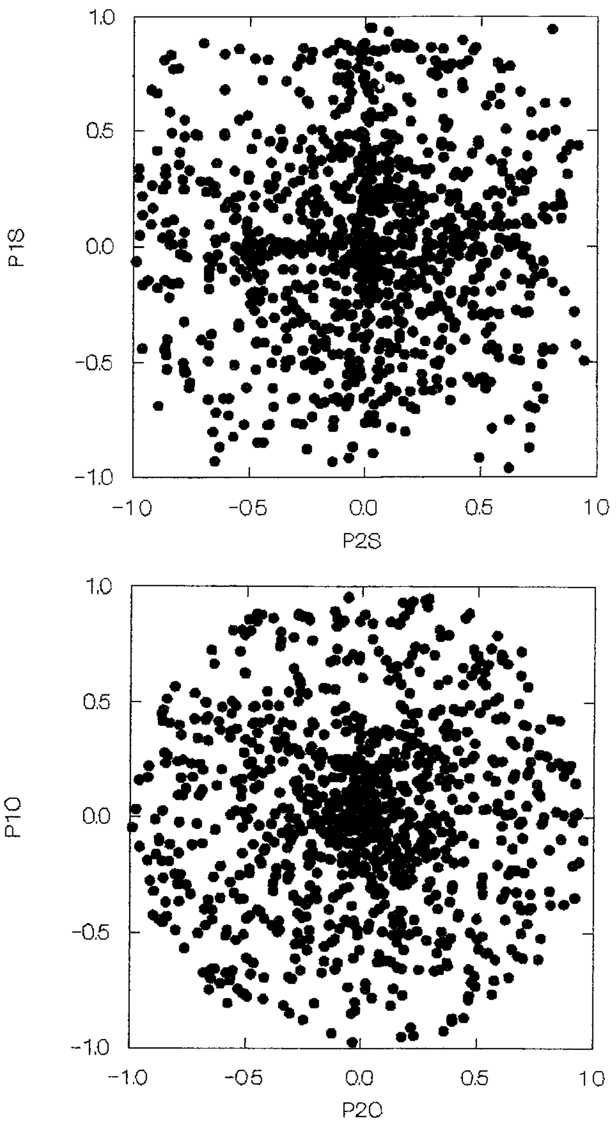

Figure 1 - Scatter Plots for $P1_O, P2_O$ and $P1_S, P2_S$.

are not the same.

If the goal is to preserve a transformed variable's "low order" statistics, and the transformation is nonlinear, in general a "high order" statistical model is required of the original variables. For the FET problem, the original variables are the FET model parameters and the transformed variables are the FET S-parameters. This is explored with the FET S-parameter data in the next section.

3.0 FET MODEL EXAMPLE

We started with 127 measured S-parameters from a .5 micron GaAs FET manufactured at TriQuint Inc. [2,3]. We duplicated these values to expand the file size to 1000 measurements. We considered only one frequency, f = 6 GHz. We labeled this data set SP_O. Using the FET model equations [4], the 1000 sets of SP's in SP_O were converted into 1000 sets of FP's, which we labeled as data set FP_O. We next created a data set, FP_S, with the same marginal PDF's and COV as FP_O. We then nonlinearly transformed the 1000 points in FP_S into 1000 SP's using the FET equations. We labeled this data set SP_S. The relationship between these data sets is illustrated in Figure 2.

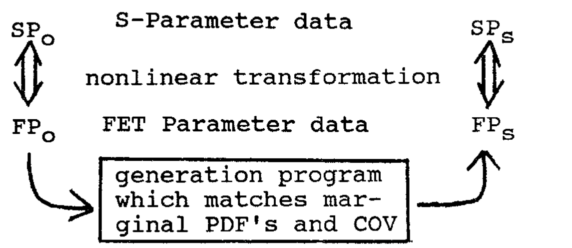

FIGURE 2 - Relation Among The Data Sets SP_O, FP_O, SP_S, FP_S.

The low order statistics of FP_O and FP_S are shown in Table 1. The low order statistics of SP_O and SP_S are shown in Table 2. Figure 3 shows the marginal PDF's for the first two parameters in the data sets FP_O and FP_S, and Figure 4 shows the same for SP_O and SP_S.

A comparison of the data in Table 1 shows the low order statistics for the FP's in FP_O and FP_S are nearly identical. The K-S test applied pairwise to the parameters in FP_O and FP_S shows a confidence of 1.000 (four significant digits) for each parameter. The PDF's are statistically identical.

A comparison of the data in Table 2 shows that the statistics for SP_O and SP_S are also similar. However the marginal PDF's shown in Figure 4 are significantly different. The K-S test applied to these densities shows a confidence of 0.001 or less for each of the eight parameters. Thus the PDF's are statistically not related, even though they are derived from statistically similar FP files.

The example shows that a knowledge of low order statistics on the FET parameters is not sufficient to characterize the low order statistics on the S-parameters. The mechanisms responsible for this are the different higher order statistics in the two files FP_O and FP_S, which were not recorded or compared, and the nonlinear transformation relating the FP's and the SP's.

5.0 CONCLUSIONS AND FUTURE WORK

A simple statistical description of the FET parameters is not sufficient if a simple S-parameter statistical description is to be faithfully captured. At present it is not known exactly what models and data modes are best. Care must be taken when collecting FET parameter statistics from S-parameter statistics to assure that the data set adequitely describes the S-parameters.

The authors are presently developing modeling concepts and valid statistical models which assure that the model parameter set adequately describes the S-parameter statistics [6,7]. A valid statistical model will extract from an S-parameter data base a set of model parameter statistics that captures the measured S-parameter statistics.

The problems illustrated here not only apply to recording FET statistics, but also to simulating FET statistics. This paper demonstrates that care must be taken when simulating FET S-parameter statistics when using the FET parameters as the statistical variables. One solution to this problem is to save the original FET model parameter data base and don't attempt to characterize it statistically. When simulating the FET, FET parameters are taken directly from the original data base with no modifications. The authors call this the "truth model" method. It has presently been implemented by EEsof [8] in LIBRA 3.0, and it has been statistically validated. This appears to be the first implementation of a statistically validated FET simulation model in commercial CAD software. The details and properties of the truth model, as well as some variations, will be presented in a future paper.

6.0 REFERENCES

[1] M. Meehan, D. Collins, "Statistical Investigation of the GaAs FET Model to Assess It's Applicability to Design Centering and Yield Estimation," EEsof

Statistical Summary for FP_O

	mean	low	high	median	standard deviation
var0	6.104	5.229	7.269	6.249	0.523
var1	463.6	342.8	609.4	476.1	64.57
var2	0.033	0.029	0.042	0.035	0.002
var3	0.110	0.100	0.119	0.110	0.005
var4	0.432	0.388	0.499	0.443	0.020
var5	0.032	0.027	0.038	0.032	0.002
var6	4.437	3.766	4.918	4.342	0.265
var7	0.091	0.074	0.127	0.100	0.015

The correlation matrix of 1000 sets is:

	0	1	2	3	4	5	6
0	1.00						
1	0.50	1.00					
2	0.06	0.59	1.00				
3	-0.85	-0.47	-0.28	1.00			
4	0.24	0.60	0.82	-0.48	1.00		
5	-0.08	-0.69	-0.17	-0.03	-0.15	1.00	
6	-0.09	0.21	0.07	0.08	0.43	-0.27	1.00
7	0.69	0.54	0.13	-0.65	0.09	-0.36	-0.42

Statistical Summary for FP_S

	mean	low	high	median	standard deviation
var0	6.104	5.229	7.269	6.249	0.523
var1	463.6	342.9	609.4	476.1	64.57
var2	0.033	0.029	0.042	0.035	0.002
var3	0.110	0.100	0.119	0.110	0.005
var4	0.432	0.388	0.499	0.443	0.020
var5	0.032	0.028	0.038	0.032	0.002
var6	4.437	3.766	4.919	4.342	0.265
var7	0.091	0.074	0.127	0.100	0.015

The correlation matrix of 1000 sets is:

	0	1	2	3	4	5	6
0	1.00						
1	0.49	1.00					
2	0.07	0.56	1.00				
3	-0.80	-0.46	-0.29	1.00			
4	0.22	0.58	0.79	-0.46	1.00		
5	-0.09	-0.67	-0.16	-0.01	-0.14	1.00	
6	-0.12	0.15	0.04	0.10	0.39	-0.24	1.00
7	0.65	0.51	0.14	-0.59	0.10	-0.34	-0.38

Table 1 - Statistical Summary for Data Files FP_O and FP_S

Statistical Summary for SP_O

	mean	low	high	median	standard deviation
var0	0.904	0.888	0.921	0.905	0.007
var1	-77.10	-84.10	-71.09	-77.60	2.402
var2	2.194	1.924	2.662	2.293	0.104
var3	117.9	114.8	120.9	117.9	1.339
var4	0.067	0.055	0.075	0.065	0.004
var5	47.55	44.65	52.01	48.32	1.576
var6	0.747	0.697	0.785	0.741	0.025
var7	-31.07	-33.80	-28.56	-31.18	1.273

The correlation matrix of 1000 sets is:

	0	1	2	3	4	5	6
0	1.00						
1	0.77	1.00					
2	-0.36	-0.64	1.00				
3	0.74	0.93	-0.49	1.00			
4	-0.05	0.10	-0.51	0.13	1.00		
5	0.17	0.54	-0.12	0.64	-0.34	1.00	
6	-0.34	-0.28	0.67	-0.26	-0.80	0.40	1.00
7	-0.53	-0.32	0.38	-0.27	-0.65	0.52	0.80

Statistical Summary for SP_S

	mean	low	high	median	standard deviation
var0	0.904	0.877	0.922	0.899	0.007
var1	-77.10	-85.68	-70.89	-78.28	2.405
var2	2.194	1.887	2.749	2.318	0.104
var3	117.9	113.0	121.5	117.3	1.333
var4	0.067	0.055	0.078	0.066	0.004
var5	47.55	42.47	52.49	47.48	1.579
var6	0.747	0.689	0.798	0.744	0.025
var7	-31.07	-34.19	-27.96	-31.07	1.275

The correlation matrix of 1000 sets is:

	0	1	2	3	4	5	6
0	1.00						
1	0.76	1.00					
2	-0.36	-0.60	1.00				
3	0.72	0.92	-0.43	1.00			
4	-0.04	0.10	-0.41	0.10	1.00		
5	0.17	0.54	-0.10	0.63	-0.36	1.00	
6	-0.33	-0.25	0.63	-0.21	-0.77	0.41	1.00
7	-0.51	-0.30	0.38	-0.22	-0.65	0.51	0.90

Table 2 - Statistical Summary for Data Files SP_O and SP_S

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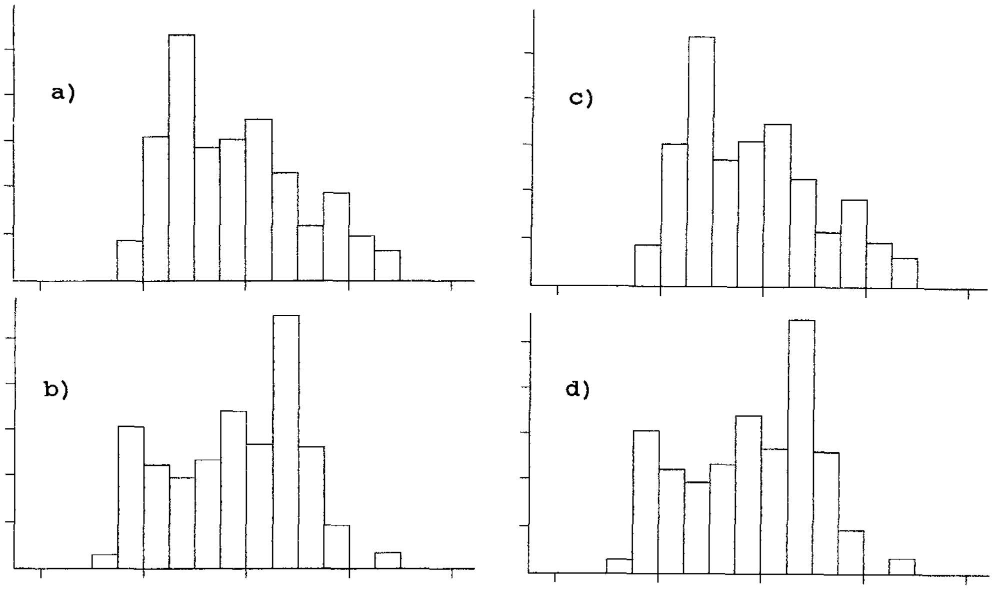

Figure 3 - Marginal Densities for a) Parameter 0 of FP_o, b) Parameter 1 of FP_o, c) Parameter 0 of FP_s, and d) Parameter 1 of FP_s.

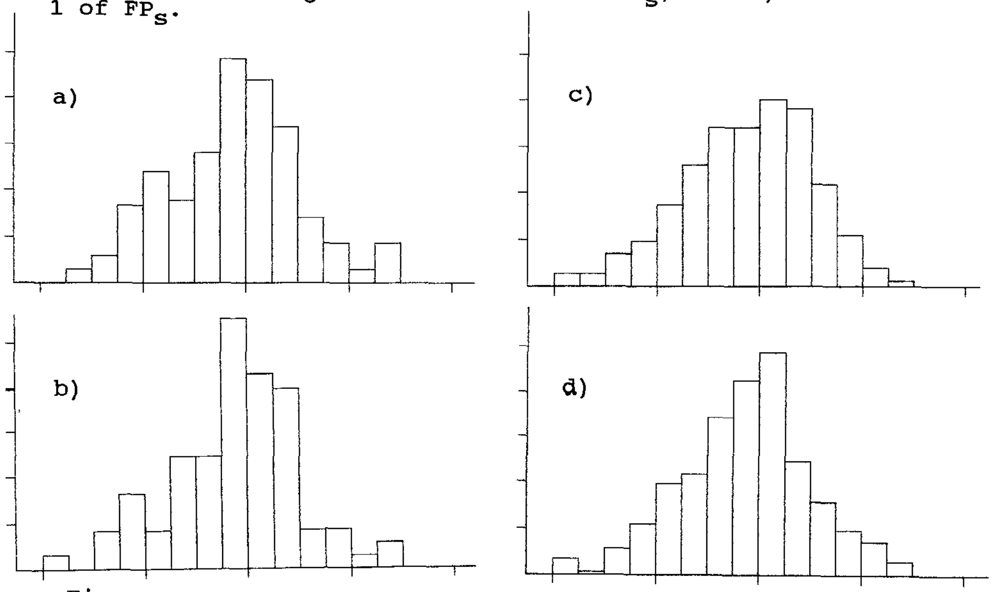

Figure 4 - Marginal Densities for a) Parameter 0 of SP_o, b) Parameter 1 of SP_o, c) Parameter 0 of SP_s, and d) Parameter 1 of SP_s.